Lecture: Exotic Financing

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Discounted Cash Flow, Section 3.6

Remark: The slightly expanded second edition (Springer, open access) has different enumeration than the first (Wiley). We use Springer's enumeration in the slides and Wiley's in the videos.

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Outline

We want to examine every conceivable finance policy. Three policies that are exotic will be presented now. They will give less general results.

We always assume no default on debt. We start with financing based on cash flows.

The idea of financing based on cash flows: the free cash flow is used for

1. debt repayments and
2. dividends.

If a **fixed portion** (extreme: all) of free cash flow goes to debt repayment, this is financing based on cash flows.

But this is difficult to treat mathematically. Hence, we simplify by assuming that **debt repayment occurs only in the first year.**

Definition 3.14: *Firm is financed based on cash flows if debt develops*

$$\widetilde{D}_t := \left(D_0 - \alpha \left(\widetilde{FCF}^{l}_1 - r_f D_0 \right) \right)^+$$

for $t \geq 1$, where $\alpha \in (0, 1]$.

To understand the definition, notice that the free cash flow minus interest is $\widetilde{FCF}^{l}_1 - r_f D_0$. From this, a fixed portion (α) is for debt repayment in the first period. A non-negative value $()^+$ is necessary because the remaining debt must not be negative.

The general valuation theorem is again very muddled. The same applies for the perpetual annuity. In particular: **valuation involves complicated derivatives** (i.e. vanilla and exotic options on shares).

Can our DCF theory be applied? There are two cases:

1. These derivatives are traded $\Longrightarrow$ DCF valuation possible.
2. These derivatives are not traded $\Longrightarrow$ DCF valuation **not** possible.

We will only verify the connection between the firm value and option pricing. But remember that using these derivatives will violate our principle from the first lesson, i.e.

Firm valuation should be done without any knowledge of stochastic structure of cash flows!

Up to now only expectations and cost of capital were necessary. This changes now. . .

For valuation of options, stochastic structure of cash flows is indispensable. This is to say that (partially) complete markets are necessary.

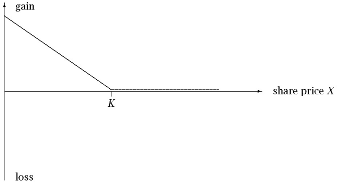
That is why we concentrate on the binomial example.

Remark (without proof): In this binomial example **any** option can be valued.

We show: company valuation will involve price of a put on a share.

Remark: A 'put of share X with exercise price K' is a bet that share price later (at expiration date) is below K.

payoff diagram at expiration date



The valuation of the put can be obtained from the **fundamental theorem**.

Today's price of the put Π is found by

1. determining expectation of payoff under Q and
2. discounting with riskless rate r_f until today.

If expiration date is $t = 1$ this translates to

$$\Pi = \frac{\mathrm{E}_Q[\text{payoff at } t = 1]}{1 + r_f} \, .$$

Payoff of a put

What is the put's payoff?

$$\text{payoff at } t = 1 \text{ is } (K - X)^+$$
$$\mathsf{E}_Q[\text{payoff at } t = 1] = \mathsf{E}_Q[(K - X)^+]$$
$$\Pi = \frac{\mathsf{E}_Q[\text{payoff at } t = 1]}{1 + r_f} = \frac{\mathsf{E}_Q[(K - X)^+]}{1 + r_f}.$$

How does this relate to the value of the levered firm?

Look at the future amount of debt,

$$\widetilde{D}_t = \left((1+\alpha r_f(1-\tau))D_0 - \alpha \widetilde{FCF}_1^{\mathrm{u}}\right)^+ .$$

Put this into our general equation (2.10),

$$V_0^{\mathrm{l}} = V_0^{\mathrm{u}} + \frac{\tau r_f D_0}{1+r_f} + \sum_{t=1}^{T-1} \frac{\tau r_f \, \mathrm{E}_Q\left[\left((1+\alpha r_f(1-\tau))D_0 - \alpha \widetilde{FCF}_1^{\mathrm{u}}\right)^+\right]}{(1+r_f)^{t+1}}.$$

The nominator does not depend on t. This can be simplified to

(well, not really 'simple')

$$V_0^{\mathrm{l}} = V_0^u + \frac{\tau r_f D_0}{1 + r_f}$$

$$+ \frac{\tau \, \mathrm{E}_Q \left[\left((1 + \alpha r_f (1 - \tau)) D_0 - \alpha \widetilde{FCF}_1^{\mathrm{u}} \right)^+ \right]}{1 + r_f} \left(1 - \frac{1}{(1 + r_f)^{T-1}} \right).$$

A problem occurs in the third summand. We cannot determine the expectation of $\left((1 + \alpha r_f (1 - \tau)) D_0 - \alpha \widetilde{FCF}_1^{\mathrm{u}} \right)^+$ without knowledge of the stochastic structure of the cash flows!

Using Theorem 3.2 (Gordon-Shapiro) the valuation equation can be modified to

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$$V_0^{\mathrm{l}} = V_0^{\mathrm{u}} + \frac{\tau r_f D_0}{1+r_f} + \\ + \underbrace{\tau \alpha d_1^u \left(1 - \frac{1}{(1+r_f)^{T-1}}\right)}_{\text{known variables}} \underbrace{\frac{\mathrm{E}_Q\left[\left(\frac{1+\alpha r_f(1-\tau)}{\alpha d_1^u} D_0 - \widetilde{V}_1^{\mathrm{u}}\right)^+\right]}{1+r_f}}_{\text{closer look necessary}}$$

Valuation equation decomposes into two parts: known variables and unknown variables.

Valuation equation – finally 13

This unknown variable

$$\Pi = \frac{\mathrm{E}_Q\left[\left(\overbrace{\frac{1+\alpha r_f(1-\tau)}{\alpha d_1^u} D_0}^{=:K} - \overbrace{\widetilde{V}_1^{\mathrm{u}}}^{=:X}\right)^+\right]}{1+r_f}$$

is also the price of a put on the share (X) with exercise price $K = \frac{1+\alpha r_f(1-\tau)}{\alpha d_1^u} D_0$.

Hence, the price of a levered firm is determined by the put because

$$V_0^{\mathrm{l}} = V_0^{\mathrm{u}} + \frac{\tau r_f D_0}{1+r_f} + \tau \alpha d_1^u \left(1 - \frac{1}{(1+r_f)^{T-1}}\right) \Pi \,.$$

The idea of financing based on dividends: managers try to hold **dividends** to be paid to shareholders **constant**. Because free cash flow is used for

1. debt repayment and
2. dividends

this has implications for debt schedule!

What can be distributed to shareholders?

$$\widetilde{FCF}^{\text{l}}_{t} - \widetilde{D}_{t-1} - \widetilde{I}_t + \widetilde{D}_t.$$

Hence

$$Div \stackrel{!}{=} \widetilde{FCF}^{\text{l}}_{t} - \widetilde{D}_{t-1} - \widetilde{I}_t + \widetilde{D}_t$$

or

$$\widetilde{D}_t \stackrel{!}{=} Div - \widetilde{FCF}^{\text{l}}_{t} + \widetilde{D}_{t-1} + \widetilde{I}_t.$$

Definition 3.15: *Financing is based on dividends over n periods if*

$$\widetilde{D}_t := \left(Div - \widetilde{FCF}^{\text{l}}_{t} + \widetilde{D}_{t-1} + \widetilde{I}_t \right)^{+}.$$

If this policy is carried out for ever, the value of the company would then be $D_0 + \frac{Div}{r_f}$. This probably violates transversality (for example: set $Div = 0!$).

Furthermore, the valuation equation again involves complicated derivatives. We do not present it here.

The dynamic leverage ratio sets cash flows in relation to firm's debts,

$$\widetilde{L}_t^d = \frac{\widetilde{D}_t}{\widetilde{FCF}_t^{\,l}}.$$

This ratio serves as a simple criterion for the time in which the company will be completely self-financed (if cash flow is used solely for debt redemption).

Definition 3.16: *Financing is based on debt-cash flow ratios if these ratios are deterministic.*

We skip the formula again...

What we have learned is:

- If the amount of debt is deterministic $\Longrightarrow$ use APV.
- If the debt ratio is deterministic $\Longrightarrow$ use WACC (FTE, TCF).
- Other financing policy requires different approaches.

In particular, **APV and WACC will not yield the same firm value** because the tax advantages from debt are on the one hand (APV) certain, and on the other hand (WACC) uncertain.

In our finite example the company's values don't differ dramatically. Is that of practical relevance? Yes, because

1. we do not know what happens if the firm lives longer and
2. we do not know what happens if cash flows are different!

And these questions cannot be answered without a proper theory.

We considered several exotic forms of financing.

The valuation now usually requires knowledge (and trading) of complicated derivatives.

The extent of value differences cannot be answered without proper theory.