# Lecture: Cost of Capital and Tax Rate

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Discounted Cash Flow, Section 4.2

Remark: The slightly expanded second edition (Springer, open access) has different enumeration than the first (Wiley). We use Springer's enumeration in the slides and Wiley's in the videos.

# Outline



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# The problem

Up to now we have looked for the firm's value **given the tax rate**.

Now we ask for a **varying tax rate** $\tau$.

This boils down to the question of how cost of capital $k^{E,\mathrm{u}}$ changes with $\tau$.

(Remember: $k^{E,\mathbf{u}}$ **is post-tax!**)

# Usually stated (Johansson 1969)

affected by the presence of investor taxes. Let $\tau_e$ be the tax rate investors pay on equity income (dividends) and $\tau_i$ be the tax rate investors pay on interest income. Then, given an expected return on debt $r_D$, define $r_D^*$ as the expected return on equity income that would give investors the same after-tax return:

$$r_D^*(1-\tau_e) = r_D(1-\tau_i)$$

So

$$r_D^* \equiv r_D \frac{(1-\tau_i)}{(1-\tau_e)} \qquad (18.23)$$

Because the unlevered cost of capital is for a hypothetical firm that is all equity,

Berk/DeMarzo: Corporate Finance, 2007

The literature on valuation suggests a relation between cost of equity post-tax $k^{E,\mathrm{u}}$ and tax rate $\tau$ where

$$k^{E,\mathrm{u}} = k^E(1-\tau). \qquad (4.6)$$

$k^E$ is sometimes interpreted as ‘cost of capital before–tax’.

Important is only the linearity: For example, increasing the tax rate from 0% to 50% lowers cost of capital by one half.

Nevertheless, **this equation is very problematic**.

# Example

Look at a company that

- lives infinitely,
- has constant expected cash flows,
- no retainments and no investments.

For such a firm

$$\widetilde{FCF}_t^{\mathrm{u}} = \widetilde{GCF}_t(1-\tau) \tag{4.5}$$

holds, which is very convenient.

(Assumptions has to be made about gross instead of free cash flows because the tax rate will change.)

# Valuation equation

Then

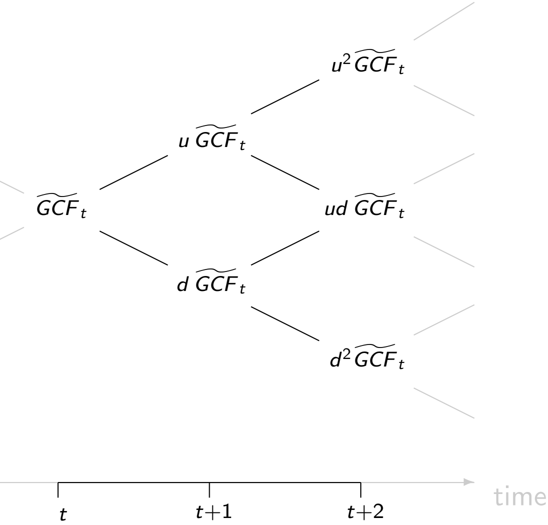
$$\widetilde{V}_t = \frac{\widetilde{FCF}_t^{\mathrm{u}}}{k^{E,\mathrm{u}}}$$

and from (4.5) with (4.6)

$$\widetilde{V}_t = \frac{\widetilde{FCF}_t^{\mathrm{u}}}{k^{E,\mathrm{u}}} = \frac{\widetilde{GCF}_t(1-\tau)}{k^E(1-\tau)} = \frac{\widetilde{GCF}_t}{k^E}\,. \quad (4.7)$$

**The personal income tax rate cancels!** Personal taxes do not seem to have an influence on company value.

# Stochastic structure of gross cash flows




Consider our infinite example with gross cash flows following up with subjective probability $P(u)$, down with $P(d)$.

# Free cash flows weak autoregressive

Gross cash flows (before tax!) are weak autoregressive.
Are free cash flows (post tax!) weak autoregressive as well?

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\begin{aligned}
\mathrm{E}\left[\widetilde{FCF}^{\mathrm{u}}_{t+1}|\mathcal{F}_t\right] &= \mathrm{E}\left[(1-\tau)\widetilde{GCF}_{t+1}|\mathcal{F}_t\right] \\
&= (1-\tau)\,\mathrm{E}\left[\widetilde{GCF}_{t+1}|\mathcal{F}_t\right] \\
&= (1-\tau)P(u)u\widetilde{GCF}_t + (1-\tau)P(d)d\widetilde{GCF}_t \\
&= \Big(\underbrace{P(u)u + P(d)d}_{:=1+g}\Big)(1-\tau)\widetilde{GCF}_t \\
&= (1+g)\widetilde{FCF}^{\mathrm{u}}_t.
\end{aligned}
$$

Yes!

# The market

Now consider **two firms**

| | firm A | firm A' |
|---|---|---|
| up and down factor | $u, d$ | $u', d'$ |
| gross cash flows | $\widetilde{GCF}_t$ | $\widetilde{GCF}'_t$ |
| firm values | $\widetilde{V}_t$ | $\widetilde{V}'_t$ |
| cost of capital | $k$ | $k'$ |
| growth rate | $g \stackrel{!}{=} 0$ | $g' \stackrel{!}{=} 0$ |

The up- and down-movements of both cash flows are perfectly correlated with probability $P(u)$ and $P(d)$.

# Duplication

There is one riskless bond with value $B_t$ at time $t$. The riskless interest rate after tax is $r_f(1-\tau)$. We now duplicate the payments of firm A' by a **portfolio of A and bond**.

This portfolio contains

$n_B :=$ bonds and

$n_A :=$ shares of firm A

such that its payments equal the dividend of A'. Or,

$$n_B B_t \left(1 + r_f(1-\tau)\right) + n_A \left(\widetilde{GCF}_{t+1}(1-\tau) + \widetilde{V}_{t+1}\right) = \widetilde{GCF}'_{t+1}(1-\tau) + \widetilde{V}'_{t+1}.$$

# Duplication

To determine $n_A$ and $n_B$ we use (4.7) and this gives

$$n_B B_t \left(1 + r_f(1-\tau)\right) + n_A \left(1 + k_{t+1}(1-\tau)\right) \widetilde{V}_{t+1} = \left(1 + k'_{t+1}(1-\tau)\right) \widetilde{V}'_{t+1}$$

or, given the stochastic structure,

$$n_B \left(1 + r_f(1-\tau)\right) B_t + n_A \left(1 + k(1-\tau)\right) u \widetilde{V}_t = \left(1 + k'(1-\tau)\right) u' \widetilde{V}'_t$$
$$n_B \left(1 + r_f(1-\tau)\right) B_t + n_A \left(1 + k(1-\tau)\right) d \widetilde{V}_t = \left(1 + k'(1-\tau)\right) d' \widetilde{V}'_t.$$

# Duplication

This is a 2×2-system that can easily be solved:

$$n_B = \frac{\widetilde{V}_t'}{B_t} \frac{(u - u')(1 + k'(1 - \tau))}{u(1 + r_f(1 - \tau))}$$
$$n_A = \frac{\widetilde{V}_t'}{\widetilde{V}_t} \frac{u'(1 + k'(1 - \tau))}{u(1 + k(1 - \tau))}.$$

(All variables are uncertain.)

Furthermore, since the market is free of arbitrage, we must have

$$n_B B_t + n_A \widetilde{V}_t = \widetilde{V}_t'.$$

# Duplication

There are now three equations. Plugging them all together results in

$$\frac{u-u'}{1+r_f(1-\tau)}+\frac{u'}{1+k(1-\tau)}=\frac{u}{1+k'(1-\tau)} \qquad (4.10)$$

and this is a relation between

- the cost of capital $k, k'$ and $r_f$ before taxes,
- the tax rate $\tau$, and
- the parameters $u$ and $u'$.

# Duplication

Equation (4.10) is a no arbitrage-condition. If it is not satisfied there is an arbitrage opportunity in the market.

But: It is also a quadratic equation and such an equation has only two solutions. These are

$\tau = 100\,\%$ and

$\tau = 0\%$.

For any other tax rate **there must be an arbitrage opportunity.** This violates our basic principle of valuation.

# Intuition of the result

Our result is in fact surprising. Is there any intuition for it?

Notice that cost of capital $k^{E,u}$ and company value $\widetilde{V}_t$ are related to each other (like "two sides of a coin"). By determining a relation between cost of capital and tax rate we implicitly determine a relation between value and tax rate.

But this relation is highly non-linear which is the reason for our arbitrage opportunity.

# Summary

**Never ever use**

$$k^{E,\mathrm{u}} = k^{E}(1-\tau)$$

**when the tax rate $\tau$ changes.**